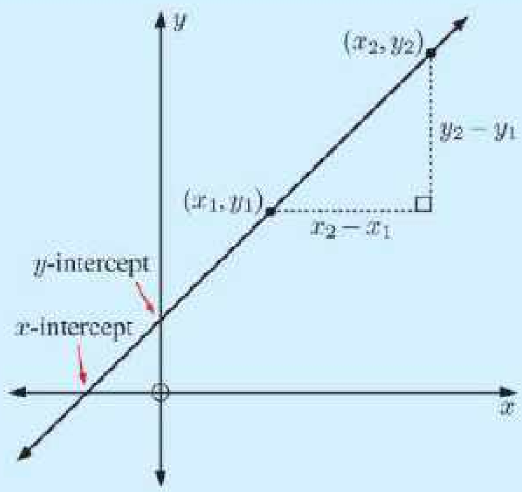
Name: ____

IB Mathematic Summer Packet Chapter 1 "Straight Lines"

In previous years you should have seen that:

- The **x-intercept** of a line is the value of x where the line cuts the x-axis.
- The **y-intercept** of a line is the value of y where the line cuts the y-axis.
- The **gradient** of a line is a measure of its steepness.
  The gradient of the line passing through (x_1, y_1) and (x_2, y_2) is $\dfrac{y\text{-step}}{x\text{-step}} = \dfrac{y_2 - y_1}{x_2 - x_1}$.
- Two lines are **parallel** if their gradients are equal.
- Two lines are **perpendicular** if their gradients are negative reciprocals of one another.



THE EQUATION OF A LINE

The **equation of a line** is an equation which connects the x and y values for every point on the line.

Using the gradient formula, the position of a general point (x, y) on a line with gradient m passing through (x_1, y_1), is given by $\dfrac{y - y_1}{x - x_1} = m$.

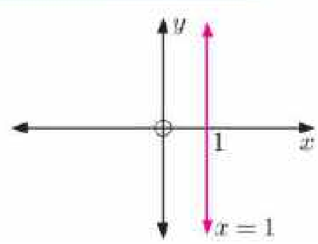
Rearranging, we find the **equation of the line** is $y - y_1 = m(x - x_1)$.

We call this **gradient-point** form. It allows us to quickly write down the equation of a line given its gradient and any point on it.

We can then rearrange the equation into other forms:

- In **gradient-intercept form**, the equation of the line with gradient m and y-intercept c is $y = mx + c$.
- In **general form**, the equation of a line is $Ax + By = C$ where A, B, C are constants.

  The general form allows us to write the equations of vertical lines, for which the gradient is undefined.
  For the line $x = 1$ we let $A = 1$, $B = 0$, and $C = 1$.



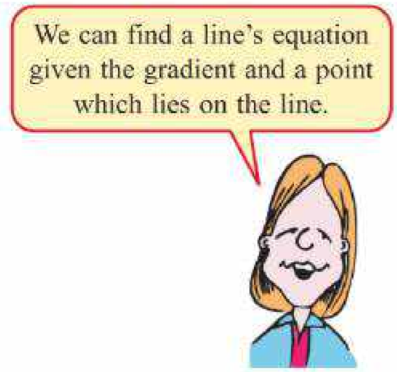
Example 1 — Self Tutor

Find, in gradient-intercept form, the equation of the line with gradient -3 that passes through $(4, -5)$.

The equation of the line is $y - (-5) = -3(x - 4)$

$\therefore \ y + 5 = -3x + 12$

$\therefore \ y = -3x + 7$



Exercise 1A

1) State the gradient and y-intercept of the line with equation:

d $y = \dfrac{7x+2}{9}$ **e** $y = \dfrac{2x-3}{6}$ **f** $y = \dfrac{3-5x}{8}$

2 Find, in gradient-intercept form, the equation of the line which has:

a gradient 3 and passes through $(4, 1)$ **b** gradient -2 and passes through $(-3, 5)$

5 The height of a helicopter above sea level t minutes after taking off is $H = 150 + 120t$ metres.

a What height above sea level did the helicopter take off from?

b Interpret the value 120 in the equation.

c Find the height of the helicopter above sea level after 2 minutes.

d How long will it take for the helicopter to be 650 m above sea level?

Example 2 **Self Tutor**

Write the equation:

a $y = -\frac{2}{3}x + 2$ in general form

b $3x - 4y = -2$ in gradient-intercept form.

a $y = -\frac{2}{3}x + 2$

$\therefore\ 3y = -2x + 6$

$\therefore\ 2x + 3y = 6$

b $3x - 4y = -2$

$\therefore\ -4y = -3x - 2$

$\therefore\ y = \frac{3}{4}x + \frac{1}{2}$

6 Write in general form:

a $y = -4x + 6$ **b** $y = 5x - 3$

7 Write in gradient-intercept form:

a $5x + y = 2$ **b** $3x + 7y = -2$

9 Match pairs of lines which are parallel:

A $y = -x + 3$ **B** $y + 2 = 3(x - 1)$ **C** $3x - y = -2$ **D** $x + y = 4$

10 Match pairs of lines which are perpendicular:

A $x + 2y = 1$ **B** $2x + y = -3$ **C** $y - 7 = 2(x + 4)$ **D** $y = 2x - 7$

Example 3 — Self Tutor

Find, in general form, the equation of the line with gradient $\frac{2}{3}$ that passes through $(-2, -1)$.

Since the line has gradient $\frac{2}{3}$, the general form of its equation is $2x - 3y = C$

Using the point $(-2, -1)$, the equation is $2x - 3y = 2(-2) - 3(-1)$

which is $2x - 3y = -1$

11 Find, in general form, the equation of the line which has:

a gradient -4 and passes through $(1, 2)$

Example 4 — Self Tutor

Find, in gradient-intercept form, the equation of the line which passes through A$(3, 2)$ and B$(5, -1)$.

The line has gradient $= \dfrac{-1-2}{5-3} = \dfrac{-3}{2} = -\dfrac{3}{2}$,

and passes through the point A$(3, 2)$.

$\therefore$ the equation of the line is

$$y - 2 = -\tfrac{3}{2}(x - 3)$$
$$\therefore\ y - 2 = -\tfrac{3}{2}x + \tfrac{9}{2}$$
$$\therefore\ y = -\tfrac{3}{2}x + \tfrac{13}{2}$$

We could use *either* A or B as the point which lies on the line.

12 Find, in gradient-intercept form, the equation of the line which passes through:

a A$(-2, 1)$ and B$(3, 11)$

b A$(7, 2)$ and B$(4, 5)$

15 Find the equation of the line which is:

a parallel to $y = 3x - 2$ and passes through $(1, 4)$

d perpendicular to $x + 2y = 6$ and passes through $(-2, -1)$.

Example 5 **Self Tutor**

a Find m given that $(-2, 3)$ lies on the line with equation $y = mx + 7$.

b Find k given that $(3, k)$ lies on the line with equation $x + 4y = -9$.

a Substituting $x = -2$ and $y = 3$ into the equation gives

$$3 = m(-2) + 7$$
$$\therefore \quad 2m = 4$$
$$\therefore \quad m = 2$$

b Substituting $x = 3$ and $y = k$ into the equation gives

$$3 + 4k = -9$$
$$\therefore \quad 4k = -12$$
$$\therefore \quad k = -3$$

17 Determine whether:

a $(3, 11)$ lies on the line with equation $y = 4x - 1$

18c) **c** Find t given that $(t, 4)$ lies on the line with equation $y = \frac{2}{3}x - \frac{4}{3}$.

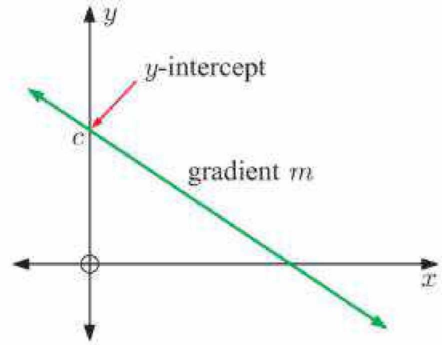
19 Find k given that:

a $(6, -3)$ lies on the line with equation $2x + 5y = k$

LINES IN GRADIENT-INTERCEPT FORM

To draw the graph of $y = mx + c$ we:

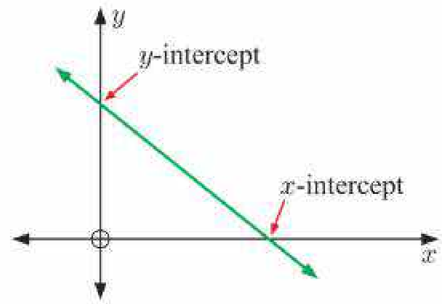
- Use the y-intercept c to plot the point $(0, c)$.
- Use x and y-steps from the gradient m to locate another point on the line.
- Join the two points and extend the line in either direction.



LINES IN GENERAL FORM

To draw the graph of $Ax + By = C$ we:

- Find the y-intercept by letting $x = 0$.
- Find the x-intercept by letting $y = 0$.
- Join the points where the line cuts the axes and extend the line in either direction.



Example 6 **Self Tutor**

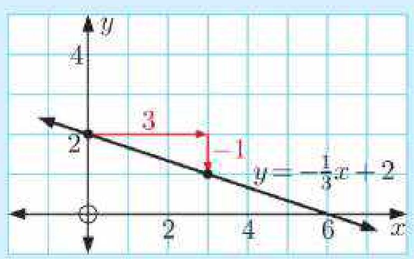
Draw the graph of:

a $y = -\frac{1}{3}x + 2$ **b** $2x - 4y = 12$

a For $y = -\frac{1}{3}x + 2$:

- the y-intercept is $c = 2$
- the gradient is

$m = \frac{-1}{3}$ ← y-step, ← x-step



b When $x = 0$, $-4y = 12$

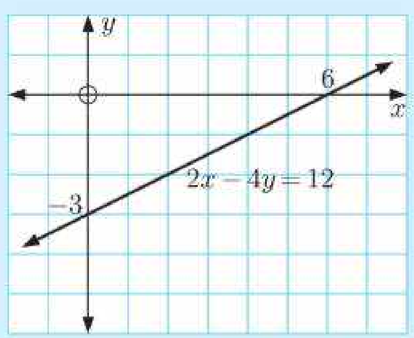
$\therefore\ y = -3$

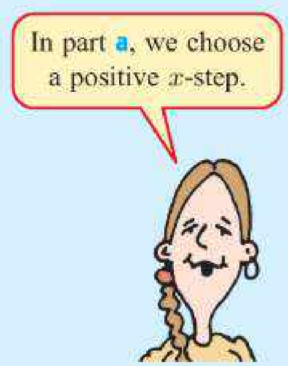
So, the y-intercept is -3.

When $y = 0$, $2x = 12$

$\therefore\ x = 6$

So, the x-intercept is 6.





Exercise 1B

2 Draw the graph of:

a $3x + 2y = 12$

d $2x + 5y = 15$

4 Consider the line with equation $2x - 3y = 18$.

a Find the axes intercepts of the line.

b Determine whether the following points lie on the line:

i $(3, -4)$ **ii** $(7, -2)$

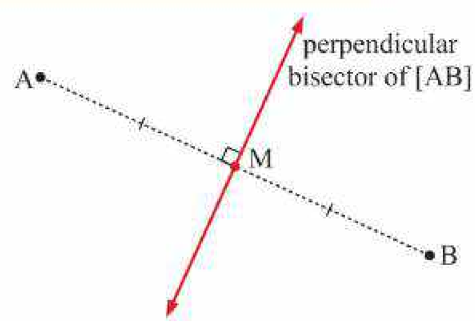
c Find c such that $(-3, c)$ lies on the line.

d Draw the graph of the line, showing the results you have found.

The **perpendicular bisector** of a line segment [AB] is the line perpendicular to [AB] which passes through its midpoint.

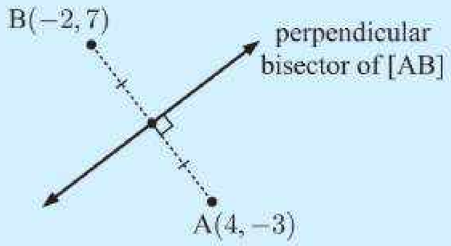
Notice that:

- Points on the perpendicular bisector are equidistant from A and B.
- The perpendicular bisector divides the number plane into two regions. On one side of the line are points that are closer to A than to B, and on the other side are points that are closer to B than to A.



Example 7 — Self Tutor

Given A(4, −3) and B(−2, 7), find the equation of the perpendicular bisector of [AB].



The midpoint M of [AB] is $\left(\frac{4+-2}{2}, \frac{-3+7}{2}\right)$

or (1, 2).

The gradient of [AB] is $\frac{7--3}{-2-4} = \frac{10}{-6} = -\frac{5}{3}$

$\therefore$ the gradient of the perpendicular bisector is $\frac{3}{5}$.

$\therefore$ the equation of the perpendicular bisector is $3x - 5y = 3(1) - 5(2)$

which is $3x - 5y = -7$.

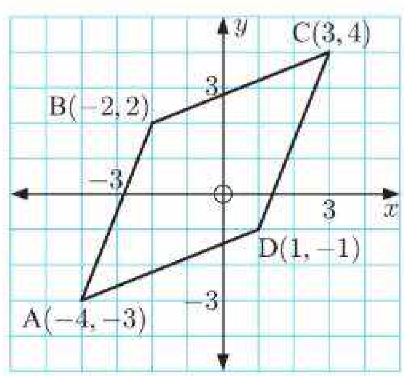
Exercise 1C

1 Consider the points A(3, 1) and B(5, 7).

- **a** Find the midpoint of [AB].
- **b** Find the gradient of [AB].
- **c** Hence state the gradient of the perpendicular bisector.
- **d** Find the equation of the perpendicular bisector.

4 Consider the quadrilateral ABCD.

- **a** Use side lengths to show that ABCD is a rhombus.
- **b** Find the equation of the perpendicular bisector of [AC].
- **c** Show that B and D both lie on this perpendicular bisector.



8 Three post offices are located in a small city at $P(-8, -6)$, $Q(1, 5)$, and $R(4, -2)$.

a Find the equation of the perpendicular bisector of:

i [PQ] **ii** [PR] **iii** [QR].

In previous years you should have seen how the intersection of two straight lines corresponds to the simultaneous solution of their equations.

A system of two equations in two unknowns can be solved by:

- **graphing** the straight lines on the same set of axes
- algebra using **substitution** or **elimination**
- technology.

To use the **equation solver function** on your calculator, you will first need to write each equation in the general form $Ax + By = C$.

GRAPHING PACKAGE

GRAPHICS CALCULATOR INSTRUCTIONS

Example 8 **Self Tutor**

Solve simultaneously: $\begin{cases} y = x - 3 \\ 2x + 3y = 16 \end{cases}$

Illustrate your answer.

$y = x - 3$ .... (1)

$2x + 3y = 16$ .... (2)

Substituting (1) into (2) gives $2x + 3(x - 3) = 16$

$\therefore\ 2x + 3x - 9 = 16$

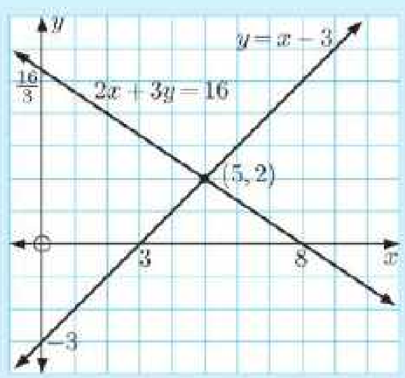
$\therefore\ 5x = 25$

$\therefore\ x = 5$

Substituting $x = 5$ into (1) gives $y = 5 - 3$

$\therefore\ y = 2$

The solution is $x = 5$, $y = 2$.



Check: Substituting into (2), $2(5) + 3(2) = 10 + 6 = 16$ ✓

Exercise 1D

1 Solve the following simultaneous equations graphically:

a $\begin{cases} y = 3x + 2 \\ y = x - 2 \end{cases}$

d $\begin{cases} y = x - 1 \\ 2x + 3y = 12 \end{cases}$

2 Solve by substitution:

a $\begin{cases} y = x + 2 \\ 2x + 3y = 21 \end{cases}$

d $\begin{cases} x = y - 3 \\ 5x - 2y = 9 \end{cases}$

Example 9 **Self Tutor**

Solve by elimination: $\begin{cases} 3x + 4y = 2 \\ 2x - 3y = 7 \end{cases}$

$3x + 4y = 2$ .... (1)
$2x - 3y = 7$ .... (2)

To make the coefficients of y the same size but opposite in sign, we multiply (1) by 3 and (2) by 4.

$$\therefore \quad 9x + 12y = 6 \quad \{(1) \times 3\}$$
$$8x - 12y = 28 \quad \{(2) \times 4\}$$

Adding, $17x = 34$

$$\therefore \quad x = 2$$

Substituting $x = 2$ into (1) gives $3(2) + 4y = 2$

$$\therefore \quad 6 + 4y = 2$$
$$\therefore \quad 4y = -4$$
$$\therefore \quad y = -1$$

The solution is $x = 2$, $y = -1$.

Check: In (2): $2(2) - 3(-1) = 4 + 3 = 7$ ✓

3 Solve by elimination:

d $\begin{cases} 3x - 7y = -27 \\ -6x + 5y = 18 \end{cases}$

i $\begin{cases} 4x - 7y = 9 \\ 5x - 8y = -2 \end{cases}$

Chapter 3 Surds and Exponents

A **radical** is any number which is written with the **radical sign** $\sqrt{}$.

A **surd** is a real, irrational radical.

For example:
- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, and $\sqrt{6}$ are surds.
- $\sqrt{4}$ is a radical but not a surd, since $\sqrt{4} = 2$.
- $\sqrt{\frac{1}{4}}$ is a radical but not a surd, since $\sqrt{\frac{1}{4}} = \frac{1}{2}$.

By definition, $\sqrt{a}$ is the non-negative number such that $\sqrt{a} \times \sqrt{a} = a$.

Notice that:
- $\sqrt{a}$ is only a real number if $a \geqslant 0$.
- For any $a \geqslant 0$, $\sqrt{a} \geqslant 0$.

A radical is in **simplest form** when the number under the radical sign is the smallest possible integer.

Example 2 — Self Tutor

Write $\sqrt{72}$ in simplest form.

$\sqrt{72}$
$= \sqrt{36 \times 2}$ {36 is the largest perfect square factor of 72}
$= \sqrt{36} \times \sqrt{2}$
$= 6\sqrt{2}$

Exercise 3A

1 Write as a single surd or rational number:

e $(3\sqrt{5})^2$ f $3\sqrt{2} \times \sqrt{5}$ g $-2\sqrt{3} \times 3\sqrt{5}$ h $2\sqrt{6} \times \sqrt{12}$

2 Write as a single surd or rational number:

e $\dfrac{\sqrt{6}}{\sqrt{18}}$ **f** $\dfrac{\sqrt{6}\times\sqrt{10}}{\sqrt{12}}$ **g** $\dfrac{\sqrt{3}}{\sqrt{6}\times\sqrt{8}}$ **h** $\dfrac{\sqrt{5}}{2\sqrt{10}\times\sqrt{2}}$

3 Write in simplest form:

a $\sqrt{12}$ **b** $\sqrt{20}$ **c** $\sqrt{27}$ **d** $\sqrt{54}$

Example 3 Self Tutor

Simplify:

a $3\sqrt{3}+5\sqrt{3}$ **b** $2\sqrt{2}-5\sqrt{2}$

a $3\sqrt{3}+5\sqrt{3}$
$= 8\sqrt{3}$

b $2\sqrt{2}-5\sqrt{2}$
$= -3\sqrt{2}$

4 Simplify:

e $3\sqrt{5}-5\sqrt{5}$ **f** $7\sqrt{3}+2\sqrt{3}$ **g** $9\sqrt{6}-12\sqrt{6}$ **h** $\sqrt{2}+\sqrt{2}+\sqrt{2}$

Example 4 Self Tutor

Simplify: $2\sqrt{75}-5\sqrt{27}$

$2\sqrt{75}-5\sqrt{27}$
$= 2\sqrt{25\times 3}-5\sqrt{9\times 3}$
$= 2\times 5\times\sqrt{3}-5\times 3\times\sqrt{3}$
$= 10\sqrt{3}-15\sqrt{3}$
$= -5\sqrt{3}$

5 Simplify:

d $2\sqrt{27}+2\sqrt{12}$ **e** $\sqrt{75}-\sqrt{12}$ **f** $\sqrt{2}+\sqrt{8}-\sqrt{32}$

Example 5 **Self Tutor**

Expand and simplify:

a $\sqrt{5}(6-\sqrt{5})$ **b** $(6+\sqrt{3})(1+2\sqrt{3})$

a
$$\sqrt{5}(6-\sqrt{5})$$
$$=\sqrt{5}\times 6+\sqrt{5}\times(-\sqrt{5})$$
$$=6\sqrt{5}-5$$

b
$$(6+\sqrt{3})(1+2\sqrt{3})$$
$$=6+6(2\sqrt{3})+\sqrt{3}(1)+\sqrt{3}(2\sqrt{3})$$
$$=6+12\sqrt{3}+\sqrt{3}+6$$
$$=12+13\sqrt{3}$$

7 Simplify:

g $(2\sqrt{2}+1)(2\sqrt{2}-1)$ **h** $(9\sqrt{3}-5)(9\sqrt{3}+5)$ **i** $(\sqrt{3}+1)^3$

Numbers like $\dfrac{6}{\sqrt{2}}$ and $\dfrac{9}{5+\sqrt{2}}$ involve division by a surd.

It is customary to "simplify" these numbers by rewriting them without the surd in the denominator.

For any fraction of the form $\dfrac{b}{\sqrt{a}}$, we can **rationalise the denominator** by multiplying by $\dfrac{\sqrt{a}}{\sqrt{a}}$.

Since $\dfrac{\sqrt{a}}{\sqrt{a}}=1$, this does not change the value of the number.

Example 7 — Self Tutor

Write with an integer denominator:

a $\frac{6}{\sqrt{5}}$ **b** $\frac{35}{\sqrt{7}}$

a

$$\frac{6}{\sqrt{5}}$$
$$= \frac{6}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$
$$= \frac{6\sqrt{5}}{5}$$

b

$$\frac{35}{\sqrt{7}}$$
$$= \frac{35}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$$
$$= \frac{{}^{5}\cancel{35}\sqrt{7}}{{}_{1}\cancel{7}}$$
$$= 5\sqrt{7}$$

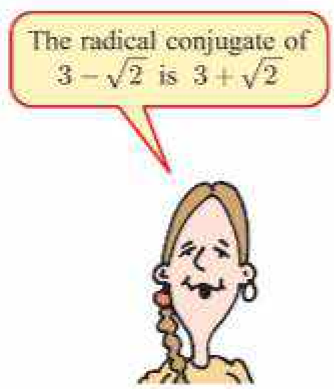
For any fraction of the form $\frac{c}{a + \sqrt{b}}$, we can remove the surd from the denominator by multiplying by $\frac{a - \sqrt{b}}{a - \sqrt{b}}$.

Expressions such as $a + \sqrt{b}$ and $a - \sqrt{b}$ are known as **radical conjugates**. They are identical except for the sign in front of the radical.

Example 8 — Self Tutor

Write $\frac{5}{3 - \sqrt{2}}$ with an integer denominator.

$$\frac{5}{3 - \sqrt{2}} = \left(\frac{5}{3 - \sqrt{2}}\right)\left(\frac{3 + \sqrt{2}}{3 + \sqrt{2}}\right)$$
$$= \frac{5(3 + \sqrt{2})}{3^2 - (\sqrt{2})^2} \quad \text{\{using the difference between two squares\}}$$
$$= \frac{15 + 5\sqrt{2}}{7}$$



Exercise 3B

1 Write with an integer denominator:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{11}{\sqrt{3}}$ **c** $\frac{\sqrt{2}}{3\sqrt{3}}$ **d** $\frac{12}{\sqrt{2}}$

i $\frac{1}{3\sqrt{5}}$ **j** $\frac{21}{\sqrt{7}}$ **k** $\frac{2}{\sqrt{11}}$ **l** $\frac{1}{(\sqrt{3})^3}$

2 Write with an integer denominator:

a $\frac{1}{3+\sqrt{2}}$ **b** $\frac{2}{3-\sqrt{2}}$ **c** $\frac{10}{\sqrt{6}-1}$ **d** $\frac{\sqrt{3}}{\sqrt{7}+2}$

i $\frac{3+\sqrt{5}}{4+\sqrt{5}}$ **j** $\frac{6-\sqrt{2}}{5-\sqrt{2}}$ **k** $\frac{\sqrt{7}+5}{\sqrt{7}-2}$ **l** $\frac{\sqrt{11}-3}{4-\sqrt{11}}$

3 Write in the form $a+b\sqrt{2}$:

a $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ **b** $\frac{5-\sqrt{2}}{6-\sqrt{2}}$ **c** $\frac{1}{(\sqrt{2}+1)^2}$ **d** $\frac{1}{(3-\sqrt{2})^2}$

NEGATIVE BASES

$(-1)^1 = -1$

$(-1)^2 = (-1) \times (-1) = 1$

$(-1)^3 = (-1) \times (-1) \times (-1) = -1$

$(-1)^4 = (-1) \times (-1) \times (-1) \times (-1) = 1$

$(-2)^1 = -2$

$(-2)^2 = (-2) \times (-2) = 4$

$(-2)^3 = (-2) \times (-2) \times (-2) = -8$

$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$

From the patterns above we can see that:

> A **negative** base raised to an **odd** power is **negative**.
> A **negative** base raised to an **even** power is **positive**.

Exercise 3C

2 Simplify, then use a calculator to check your answer:

- **a** $(-1)^5$
- **c** $(-1)^{14}$
- **d** $(-1)^{19}$
- **f** -1^8
- **g** $-(-1)^8$
- **i** -2^5
- **j** $-(-2)^6$
- **l** $-(-5)^4$

4 Use your calculator to evaluate the following. Comment on your results.

- **a** 9^{-1} and $\frac{1}{9^1}$
- **b** 6^{-2} and $\frac{1}{6^2}$

Using arguments like this, we arrive at the **laws of exponents** for $m, n \in \mathbb{Z}$:

$a^m \times a^n = a^{m+n}$	To **multiply** numbers with the **same base**, keep the base and **add** the exponents.
$\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$	To **divide** numbers with the same base, keep the base and **subtract** the exponents.
$(a^m)^n = a^{m \times n}$	When **raising** a **power** to a **power**, keep the base and **multiply** the exponents.
$(ab)^n = a^n b^n$	The power of a product is the product of the powers.
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad b \neq 0$	The power of a quotient is the quotient of the powers.
$a^0 = 1, \quad a \neq 0$	Any non-zero number raised to the power zero is 1.

$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ and in particular $a^{-1} = \frac{1}{a}, \quad a \neq 0.$

Example 9 Self Tutor

Simplify:

a $7^4 \times 7^5$ **b** $\frac{5^6}{5^3}$ **c** $(x^3)^k$

a $7^4 \times 7^5$
$= 7^{4+5}$
$= 7^9$

b $\frac{5^6}{5^3}$
$= 5^{6-3}$
$= 5^3$

c $(x^3)^k$
$= x^{3\times k}$
$= x^{3k}$

1 Simplify:

a $k^4 \times k^2$ **b** $c^8 \times c^m$ **c** $r^2 \times r^5 \times r^4$

g $(7^6)^d$ **h** $(m^3)^t$ **i** $(11^x)^{2y}$

m $(j^4)^{3x}$ **n** $\frac{z^7}{z^{4t}}$ **o** $(13^c)^{5d}$

Example 10 Self Tutor

Simplify using the laws of exponents:

a $4x^3 \times 2x^6$ **b** $\frac{15t^7}{3t^5}$ **c** $\frac{k^2 \times k^6}{(k^3)^2}$

a $4x^3 \times 2x^6$
$= 4 \times 2 \times x^3 \times x^6$
$= 8 \times x^{3+6}$
$= 8x^9$

b $\frac{15t^7}{3t^5}$
$= \frac{15}{3} \times t^{7-5}$
$= 5t^2$

c $\frac{k^2 \times k^6}{(k^3)^2}$
$= \frac{k^{2+6}}{k^{3\times 2}}$
$= \frac{k^8}{k^6}$
$= k^2$

Exercise 3D

2 Simplify using the laws of exponents:

a $\frac{4b^5}{b^2}$ **b** $2w^4 \times 3w$ **c** $\frac{12p^4}{3p^2}$ **d** $5c^7 \times 6c^4$

i $5s^2t \times 4t^3$ **j** $\frac{(k^4)^5}{k^3 \times k^6}$ **k** $\frac{12x^2y^5}{8xy^2}$ **l** $\frac{(b^3)^4 \times b^5}{b^2 \times b^6}$

Example 11 — Self Tutor

Write as a power of 2:

a 16 **b** $\frac{1}{16}$ **c** 1 **d** 4×2^n **e** $\frac{2^m}{8}$

a $16 = 2 \times 2 \times 2 \times 2 = 2^4$

b $\frac{1}{16} = \frac{1}{2^4} = 2^{-4}$

c $1 = 2^0$

d $4 \times 2^n = 2^2 \times 2^n = 2^{2+n}$

e $\frac{2^m}{8} = \frac{2^m}{2^3} = 2^{m-3}$

3 Write as a power of 2:

a 4 **b** $\frac{1}{4}$ **c** 8 **d** $\frac{1}{8}$ **e** 32 **f** $\frac{1}{32}$

5 Write as a single power of 2:

a 2×2^a **b** 4×2^b **c** 8×2^t **d** $(2^{x+1})^2$ **e** $(2^{1-n})^{-1}$

f $\frac{2^c}{4}$ **g** $\frac{2^m}{2^{-m}}$ **h** $\frac{4}{2^{1-n}}$ **i** $\frac{2^{x+1}}{2^x}$ **j** $\frac{4^x}{2^{1-x}}$

Example 12 | **Self Tutor**

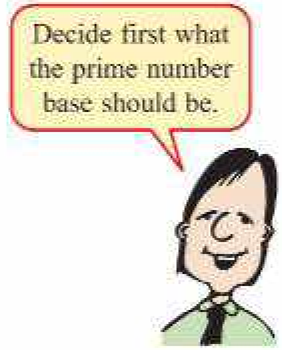
Express in exponent form with a prime number base:

a 9^4 **b** $\frac{3^x}{9^y}$ **c** 25^x

a $9^4 = (3^2)^4 = 3^{2\times4} = 3^8$

b $\frac{3^x}{9^y} = \frac{3^x}{(3^2)^y} = \frac{3^x}{3^{2y}} = 3^{x-2y}$

c $25^x = (5^2)^x = 5^{2x}$



7 Express in exponent form with a prime number base:

a 32 **b** 25^3 **c** 16^p **d** $5^a \times 25$

i 32^{2-r} **j** $\frac{81}{3^{y+1}}$ **k** $\frac{16^k}{4^k}$ **l** $\frac{5^{a+1}\times 125}{25^{2a}}$

Example 13 | **Self Tutor**

Simplify:

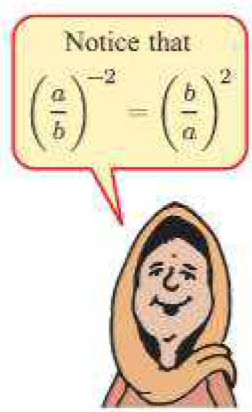
a 7^0 **b** 3^{-2} **c** $3^0 - 3^{-1}$ **d** $\left(\frac{5}{3}\right)^{-2}$

a $7^0 = 1$

b $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

c $3^0 - 3^{-1} = 1 - \frac{1}{3} = \frac{2}{3}$

d $\left(\frac{5}{3}\right)^{-2} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$



8 Simplify:

a 4^0 **b** 3^{-1} **c** 7^{-2} **d** x^{-3}

e $5^0 + 5^{-1}$ **f** $\left(\frac{5}{3}\right)^0$ **g** $\left(\frac{7}{4}\right)^{-1}$ **h** $\left(\frac{1}{6}\right)^{-1}$

i $\left(\frac{4}{3}\right)^{-2}$ **j** $2^1 + 2^{-1}$ **k** $\left(1\frac{2}{3}\right)^{-3}$ **l** $5^2 + 5^1 + 5^{-1}$

9 Write using powers of 2, 3, and/or 5:

e $\frac{4}{27}$ **f** $\frac{2^c}{8 \times 9}$ **g** $\frac{9^k}{10}$ **h** $\frac{6^p}{75}$

Example 14 **Self Tutor**

Write in simplest form, without brackets:

a $\left(-3a^2\right)^4$ **b** $\left(-\frac{2a^2}{b}\right)^3$

a $\left(-3a^2\right)^4$
$= (-3)^4 \times (a^2)^4$
$= 81 \times a^{2\times4}$
$= 81a^8$

b $\left(-\frac{2a^2}{b}\right)^3$
$= \frac{(-2)^3 \times (a^2)^3}{b^3}$
$= \frac{-8a^6}{b^3}$

These have the form $(ab)^n = a^n b^n$ or $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

10 Write without brackets:

a $(2a)^2$ **b** $(3n)^2$ **g** $\left(\frac{p}{q}\right)^4$ **h** $\left(\frac{t}{5}\right)^2$

11 Write in simplest form, without brackets:

i $\left(\frac{xy}{2}\right)^3$ **j** $\left(\frac{-2a^2}{b^2}\right)^3$ **k** $\left(\frac{-4a^3}{b}\right)^2$ **l** $\left(\frac{-3p^2}{q^3}\right)^2$

12 Expand the brackets and write in simplest form:

g $x^{-1}(x^3 + x^2 - x)$ **h** $(x^2 + x^{-1})^2$ **i** $(x^2 + x^{-1})(x^2 - x^{-1})$

Example 15 Self Tutor

Write without negative exponents:

a $(2c^3)^{-4}$ **b** $\frac{a^{-3}b^2}{c^{-1}}$

a $(2c^3)^{-4} = \frac{1}{(2c^3)^4}$

$= \frac{1}{2^4 c^{3\times 4}}$

$= \frac{1}{16c^{12}}$

b $a^{-3} = \frac{1}{a^3}$ and $\frac{1}{c^{-1}} = c^1$

$\therefore \frac{a^{-3}b^2}{c^{-1}} = \frac{b^2 c}{a^3}$

13 Write without negative exponents:

a ab^{-2} **b** $(ab)^{-2}$ **c** $(2ab^{-1})^2$ **d** $(5m^2)^{-2}$

e $(3a^{-2}b)^2$ **f** $(3xy^4)^{-3}$ **g** $\frac{a^2b^{-1}}{c^2}$ **h** $\frac{a^2b^{-1}}{c^{-2}}$

Example 16 — Self Tutor

Write $\dfrac{1}{2^{1-n}}$ without a fraction.

$$\frac{1}{2^{1-n}} = 2^{-(1-n)} = 2^{-1+n} = 2^{n-1}$$

14 Write without a fraction:

a $\dfrac{1}{a^n}$ **b** $\dfrac{5}{a^m}$ **c** $\dfrac{1}{b^{-n}}$ **d** $\dfrac{1}{2^{n-3}}$

15 Write without fractions:

e $\dfrac{1}{x}+\dfrac{3}{x^2}$ **f** $\dfrac{4}{x}-\dfrac{5}{x^3}$ **g** $7x-\dfrac{4}{x}+\dfrac{5}{x^2}$ **h** $\dfrac{3}{x}-\dfrac{2}{x^2}+\dfrac{5}{x^4}$

Example 17 — Self Tutor

Write without fractions:

a $\dfrac{x^2+3x+2}{x}$ **b** $\dfrac{2x^5+x^2+3x}{x^{-2}}$

a
$$\frac{x^2+3x+2}{x} = \frac{x^2}{x}+\frac{3x}{x}+\frac{2}{x} = x+3+2x^{-1}$$

b
$$\frac{2x^5+x^2+3x}{x^{-2}} = \frac{2x^5}{x^{-2}}+\frac{x^2}{x^{-2}}+\frac{3x}{x^{-2}} = 2x^{5-(-2)}+x^{2-(-2)}+3x^{1-(-2)} = 2x^7+x^4+3x^3$$

16 Write without fractions:

a $\frac{x+3}{x}$

b $\frac{3-2x}{x}$

c $\frac{5-x}{x^2}$

d $\frac{x+2}{x^3}$

i $\frac{5-x-x^2}{x}$

j $\frac{8+5x-2x^3}{x}$

k $\frac{16-3x+x^3}{x^2}$

l $\frac{5x^4-3x^2+x+6}{x^2}$

17 Write without fractions:

e $\frac{x^2+x-4}{x^{-2}}$

f $\frac{x^3-3x+6}{x^{-3}}$

g $\frac{x^3-6x+10}{x^{-2}}$

h $\frac{x^2+4+x^{-1}}{x^{-3}}$

Scientific notation or **standard form** involves writing any given number as a number between 1 inclusive and 10, multiplied by a power of 10.

The result has the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$.

Example 18 Self Tutor

Express in scientific notation:

a 37 600 b 0.000 008 6

a $37\,600 = 3.76 \times 10\,000$ {shift decimal point 4 places to the left and $\times$ 10 000}

$= 3.76 \times 10^4$

b $0.000\,008\,6 = 8.6 \div 10^6$ {shift decimal point 6 places to the right and $\div$ 1 000 000}

$= 8.6 \times 10^{-6}$

Exercise 3E

2 Write in scientific notation:

a 259

b 259 000

c 2 590 000 000

j 407 000

k 407 000 000

l 0.000 040 7

Example 19 Self Tutor

Write as an ordinary number:

a 3.2×10^2 b 5.76×10^{-5}

a 3.2×10^2

$= 3.20 \times 100$

$= 320$

b 5.76×10^{-5}

$= 000005.76 \div 10^5$

$= 0.000\,057\,6$

4 Write as an ordinary number:

a 4×10^3 **b** 3.8×10^5 **c** 8.6×10^1 **d** 4.33×10^7

e 4×10^{-3} **f** 3.8×10^{-5} **g** 8.6×10^{-1} **h** 4.33×10^{-7}

5 Write as a decimal number:

a It is estimated that the population of the world in 2020 will be 7.4×10^9 people.

b The mass of a Ryukyu mouse is 1.12×10^{-2} kg.

c The bacterium *bordetella pertussis* is about 5×10^{-7} m long.

d The Eiffel tower in Paris weighs approximately 7.3×10^6 kg.

10 The closest distance between Earth and Venus in their orbits is about 3.8×10^9 m.
The closest distance between Venus and Mercury in their orbits is about 7.7×10^9 m.
A spacecraft is sent from Earth to Venus, waits for the opportune time, then travels on to Mercury.

a Find the minimum distance the spacecraft must travel.

b Discuss the assumptions you have made in your answer.

11 In a vacuum, light travels at approximately $2.9979 \times 10^8\ \text{m s}^{-1}$.

a How far will light travel in:

i 1 minute **ii** 1 day?

b Assuming a year $\approx$ 365.25 days, calculate one **light-year**, the distance light will travel in a vacuum in a year.