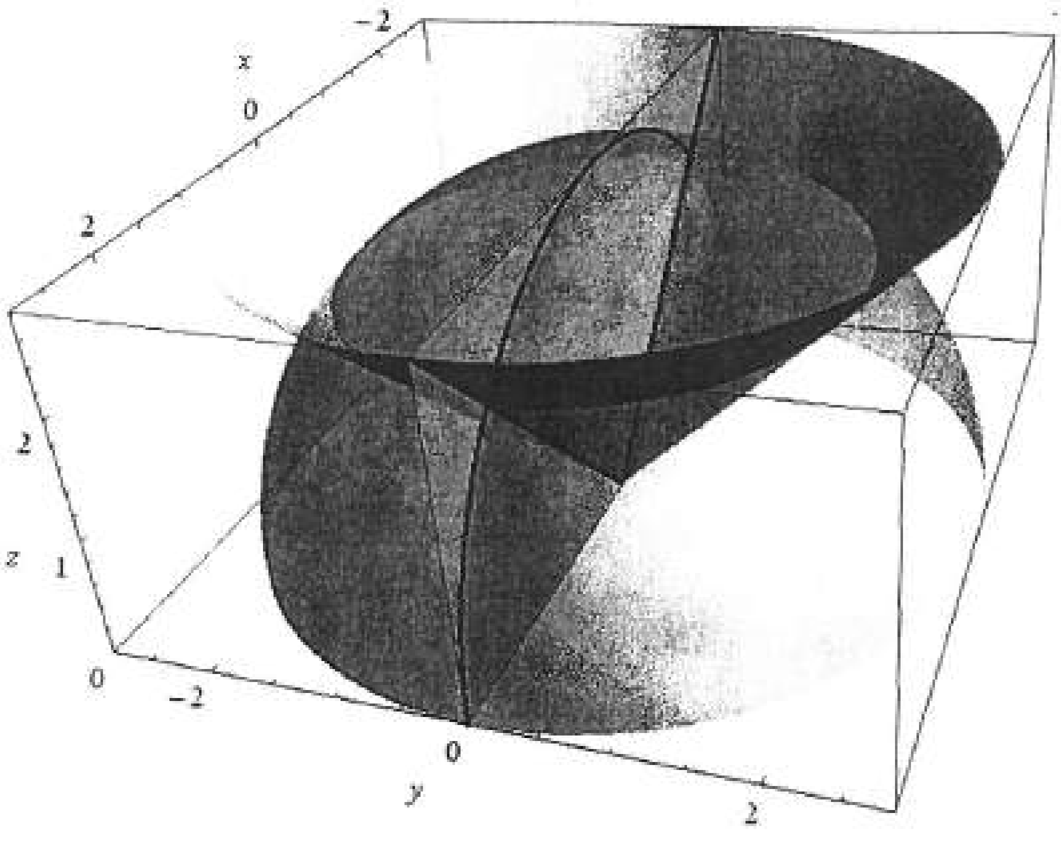
Name: ______________________

AP Calculus AB Summer Packet Mr. Hermiller



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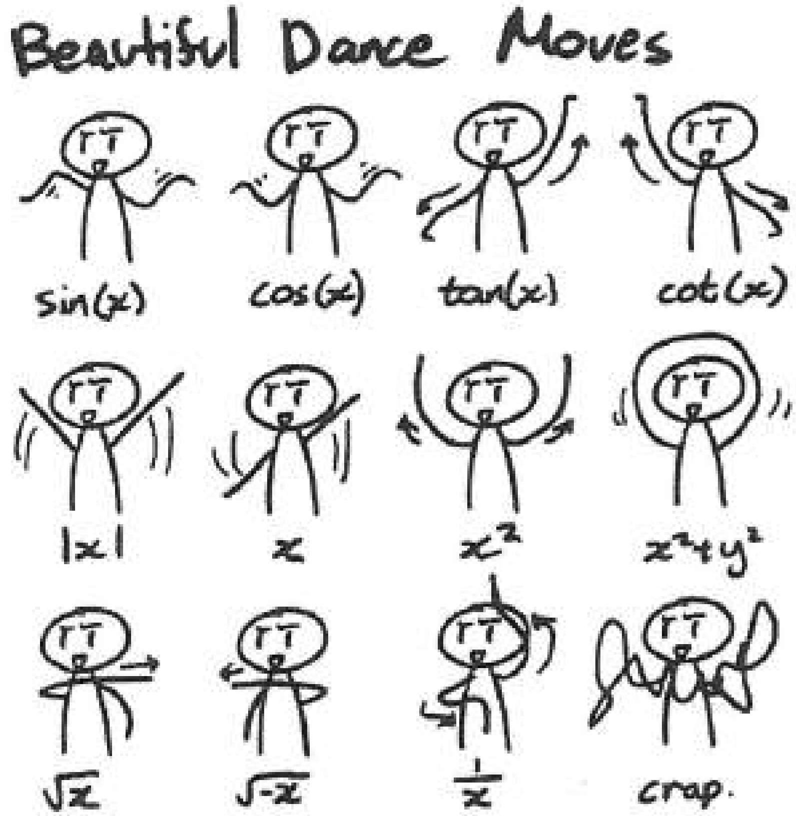
To infinity...

$$\lim_{x \to \infty}$$

... and beyond!

$$\lim_{x \to \infty +}$$



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AP CALCULUS AB: SUMMER PACKET INFORMATION PAGE

STEP 1: When you return your books at the end of the year, ask for a copy of the AP Calculus book. Let the bookroom workers know that you are checking it out to do the required summer work for the course.

STEP 2: You should have received a syllabus from Mr. Hermiller when you picked up this packet. Familiarize yourself with the expectations of the course. You may also email Mr. Hermiller at jhermiller@wscloud.org and request a copy of the syllabus.

STEP 3: You will be working your way through the first 3 sections of chapter P (Precalculus). Read each section carefully making sure that you understand all of the terms that are listed.

STEP 4: Do the problems in the packet. There is a section on Linear Equations and Models, Functions and Graphing, a section on logarithms and exponentials, and a section on Trigonometry. Complete all the problems in these sections of the packet, they will be collected for a grade when school resumes. There are additional problems listed below for each section of Chapter P. These will be collected. You are expected to do them and understand them (there are pages in this packet for the answers).

STEP 5: When you come back to school, this will be the schedule for the first six days.

Day 1: Go over expectations of the course, review
Day 2: Review
Day 3: Test over linear equations and functions. 100 pts
Day 4: Review
Day 5: Test over logarithms. 100 pts
Day 6: Review
Day 7. Test over Trigonometry. 100 pts

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AP CALCULUS AB: SUMMER PACKET TOPIC: LINEAR EQUATIONS

Linear Equations:

There are three main forms of a linear equation we will be dealing with this coming year.

Slope-intercept form: $y = mx + b$, here m is the slope of the line, and b is the y-intercept.

Point-slope form: $y - y_1 = m(x - x_1)$, here m is the slope of the line, and (x_1, y_1) is any point on the line. This format seems to be used most on the AP exam.

Standard form: $Ax + By = C$, here A, B, and C can be used to find slope $\left(m = \frac{-A}{B}\right)$, the y-intercept $\left(0, \frac{C}{B}\right)$, and the x-intercept $\left(\frac{C}{A}, 0\right)$.

Slope:

The slope of a nonvertical line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{y_1 - y_2}{x_1 - x_2}$$

If m is positive, then the line rises from left to right.
If m is negative, then the line falls from left to right.
If m is zero, then the line is horizontal. The equation of a horizontal line is $y = b$.
If m is undefined, then the line is vertical. The equation of a vertical line is $x = a$.
Lines with the same slope are parallel.
Line with slopes that are opposite reciprocals are perpendicular. ie: $m_1 = \frac{3}{4}, m_2 = \frac{-4}{3}$.

Try the following examples.

1. ____________ What is the slope of the line passing through the points $(2,3)$ and $(-2,9)$?
2. ____________ What is the slope of the line passing through the points $(4,-5)$ and $(-2,-5)$?
3. ____________ Write the equation of the line with slope $m = \frac{4}{3}$ and y-intercept $(0,-2)$.
4. ____________ Write the equation of the line that has slope $m = -3$ and passes through the point $(-2,5)$.
5. ____________ Write the equation of the line that passes through the points $(4,-1)$ and $(8,6)$.
6. ____________ Write the equation of the line passing through the points $(3,1)$ and $(3,-1)$.
7. ____________ Write the equation of the line that is parallel to the line $y = 4x - 3$ and passing though the point $(4,3)$.
8. ____________ Find the slope, y-intercept, and x-intercept of the line $5x - 3y = -15$.
9. ____________ What is the slope of a line perpendicular to the line $6x + 4y = 12$?
10. ____________ Write the equation of the line parallel to $Ax + By = C$ and passing through the point (D,E).

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Ratios and Rates of Change:

The slope of a line can be interpreted as either a *ratio* or a *rate of change*. If the x-axis and y-axis have the same unit of measure, the slope has no units and is a **ratio**. If the x-axis and y-axis have different units of measure, the slope is **rate of change**.

EX1. In a water-ski jumping tournament, the rises to a height of 6 feet on a raft that is 21 feet long. The slope of the ramp would be a ratio: $m = \frac{6\,feet}{21\,feet} = \frac{2}{7}$. The units cancel.

EX2. The population of Arizona was 1,775,000 in 1970 and was 2,718,000 in 1980. Over this 10 year period, the average rate of change in the population was:

$$m = \frac{\Delta P}{\Delta t} = \frac{change\ in\ population}{change\ in\ time} = \frac{2,718,000 - 1,775,000}{1980 - 1970} = 94,300\ people\ per\ year$$

Linear Models:

The rate of change in a story problem is always the slope of the linear model. The starting or initial amount, or the amount at time zero, is the y-intercept.

EX3. A company reimburses its sales representative \$150 per day for lodging and meals plus \$0.30 per mile driven. Write a linear model giving the daily cost C to the company in terms of x, the total miles driven that day.
Here the slope is 0.30, because that cost per mile driven or change in the cost, and the starting amount of \$150 is the y-intecept. So, the linear model is $C = 0.30x + 150$.

In some linear models, you must find the rate of change.

EX4. In example 2 above, the slope was found to be 94,300 people per year. Here is the equation:
$P - 2,718,000 = 94,300(t - 1980)$ or $P - 1,775,000 = 94,300(t - 1970)$ using point-slope form of a linear equation.

Try the following examples.

11. A cell phone customer pays a service charge of \$25.50 each month and an additional \$0.20 per minute of usage on the cell phone during that month. Write a linear model for the total monthly cost C in terms of the number of minutes x. Answer:________________

12. A small business purchases a piece of equipment for \$825. After five years, the equipment will be outdated, having no value. Write a linear model for the value V of the equipment in terms of the time t in years, where $0 \le t \le 5$. Answer:________________

13. A balloon ascends straight up into the sky. After 5 seconds it is 20 feet above the ground, and after 15 seconds it is 60 feet above the ground. Write a linear model for the altitude of the balloon A in terms of the time t in seconds. Answer:________________

14. After two years on the job, Julie was making \$11.75. Now that she has been on the job for five years, she is making \$13.25. Write a linear model for her pay P in terms of the number of years t that she has been on the job. Answer:________________

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Linear Regressions and Data:
When fitting a linear model to data, we are going to take the easy route. We are going to use our graph calculators.

EX1. The data in the table below gives the Brinell hardness H of 0.35 carbon steel when hardened and tempered at temperature t (degrees Fahrenheit).

t	200	400	600	800	1000	1200	Put this data in L_1 in your calculator.
H	534	495	415	352	269	217	Put this data in L_2 in your calculator.

To find the line of regression:
Go to STAT, then CALC, select #4, LinReg(ax+b) will pop up on your home screen. Hit enter. Your result will be $H = -0.332t + 612.933$. (We round to three decimal places in this class.) The Brinell hardness of 0.35 carbon steel at a temperature of $0°F$ is 612.933. The hardness decreases 0.332 per degree Fahrenheit.

To find the line of regression and store it in Y_1 so that you can graph it:
Go to STAT, then CALC, select #4, LinReg(ax+b) will pop up on your home screen. After LinReg(ax+b), hit VARS, move right to Y-VARS, hit #1, hit #1 again, and Y_1 pops up. When you hit enter, it finds the line of regression and stores it into Y_1.

To find the average rate of change.
You do not have to find all five slopes in this problem and average them. You could, but all you have to do is take the first and last entries, and the find the slope using those points.

$m = \frac{217 - 534}{1200 - 200} = -0.317$, which is the same as averaging all the slopes.

This slope is not the same as the regression because the data is not perfectly linear.

Try the following example.

15. The data in the table gives the variable costs for operating an automobile in the United States for the years 1985 through 1991. The functions y_1, y_2, and y_3 represent the costs in cents per mile for gas and oil, maintenance, and tires. Let x be the number of years after 1985.

Year	1985 ($x = 0$)	1986	1987	1988	1989	1990	1991
y_1 *(G&O)*	6.16	4.48	4.80	5.20	5.20	5.40	6.70
y_2 *(M)*	1.23	1.37	1.60	1.60	1.90	2.10	2.20
y_3 *(T)*	0.65	0.67	0.80	0.80	0.80	0.90	0.90

A. Use your calculator to find a linear model for y_1, y_2, and y_3. In order to use a linear regression with lists other than L_1 and L_2, you must type the names of the list right after LinReg(ax+b). ie: LinReg(ax+b) L_1,L_3 ie: LinReg(ax+b) L_1,L_3,Y_1 if you want to graph it.

y_1 = ______________________________

y_2 = ______________________________

y_3 = ______________________________

B. ____________ What is the average rate of change in the cost of maintenance from 1985 to 1991?

C. ____________ If the pattern continues, what would be the cost in cents per mile for maintenance in 1998?

D. Which model does not seem to be linear? Justify your answer.

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AP CALCULUS AB: SUMMER PACKET TOPIC: FUNCTIONS

Functions:

A **relation** between two sets X and Y is a set of ordered pairs (x,y) where x is a member of the set X, and y is a member of the set Y. A **function** from X to Y is a relation between X and Y that has the property that any two order pairs with the same x-value also have the same y-value. If a vertical line passes through the graph of a relation more than once, the graph is not a function. This is called the **vertical line test.** The x-value is called the **independent variable.** The y-value is called the **dependent variable**. The **domain** is the set X. The **range** is the set Y. A **one-to-one function** from X to Y is a relation between X and Y that has the property that any two order pairs with the same x-value also have the same y-value, and any two order pairs with the same y-value also have the same x-value. If a horizontal line passes through the graph of a relation more than once, the graph is not one-to-one. This is called the **horizontal line test.** To try the horizontal line test, the vertical line test must first pass. All relations have inverses. All functions have inverses. Any one-to-one function has an inverse that is a function.

A function $f(x)$ is said to be **negative** on an open interval $I = (x_1, x_2)$ if for any real number c on I $f(c) < 0$. A function $f(x)$ is said to be **positive** on an open interval $I = (x_1, x_2)$ if for any real number c on I $f(c) > 0$. A function $f(x)$ is said to be **increasing** on an open interval $I = (x_1, x_2)$ if for any two real numbers c and d on I, if $c < d$ then $f(c) < f(d)$. ie: the graph goes uphill from left to right on the entire interval I. A function $f(x)$ is said to be **decreasing** on an open interval $I = (x_1, x_2)$ if for any two real numbers c and d on I, if $c < d$ then $f(c) > f(d)$. ie: the graph goes downhill from left to right on the entire interval I.

EX. Here is a function written implicitly, in other words, not solved for y.

$x^2 + y = 2$

Solve it for y and the equation is said to be explicit, and we can use function notation.

$y = x^2 - 2$ or $f(x) = x^2 - 2$

In the following examples, use your calculator to graph and fill in the information. Round to three decimal places, if needed.

1. Graph the following function, and answer the corresponding questions.

A. $f(x) = x^2 - 4$

Domain: ____________

Range: ____________

1-1?: ____________

Odd, Even, or neither? ________

x-intercept(s) ____________

y-intercept(s) ____________

Intervals for x where the graph is

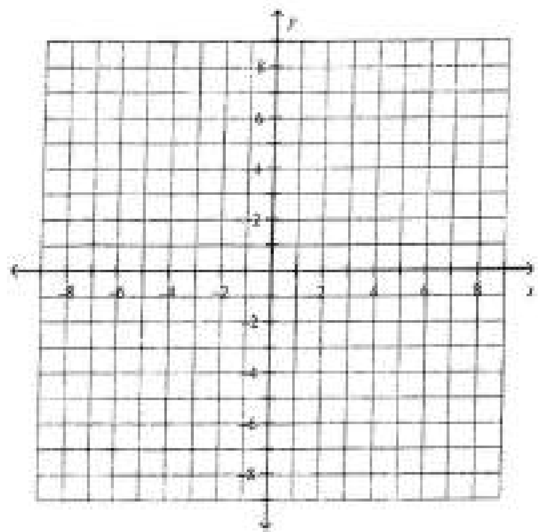
Increasing: ____________

Decreasing: ____________

Positive: ____________

Negative: ____________

$f(-3)$ = ____________



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2. Graph the following function, and answer the corresponding questions.

A. $f(x) = 2x^3 - 15x^2 - 36x + 2$

Domain: ______________

Range: ______________

1-1?: ______________

Odd, Even, or neither? ________

x-intercept(s) ______________

y-intercept(s) ______________

Intervals for x where the graph is

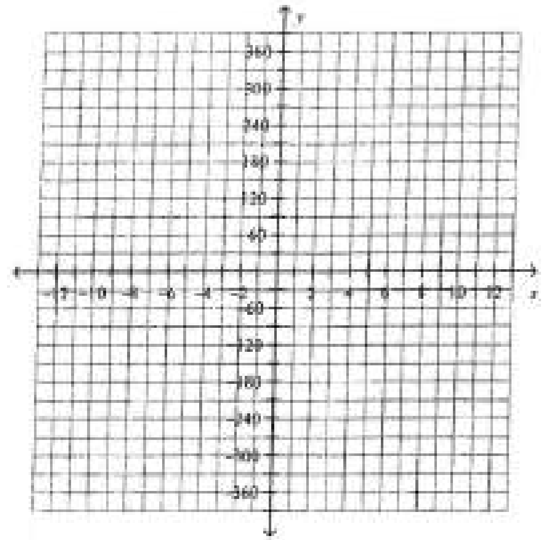
Increasing: ______________

Decreasing: ______________

Positive: ______________

Negative: ______________

$f(-2) =$ ______________



3. Graph the following function, and answer the corresponding questions.

A. $f(x) = \dfrac{x-2}{x^2-9}$

Domain: ______________

Range: ______________

1-1?: ______________

Odd, Even, or neither? ________

x-intercept(s) ______________

y-intercept(s) ______________

Asymptotes: ______________

Intervals for x where the graph is

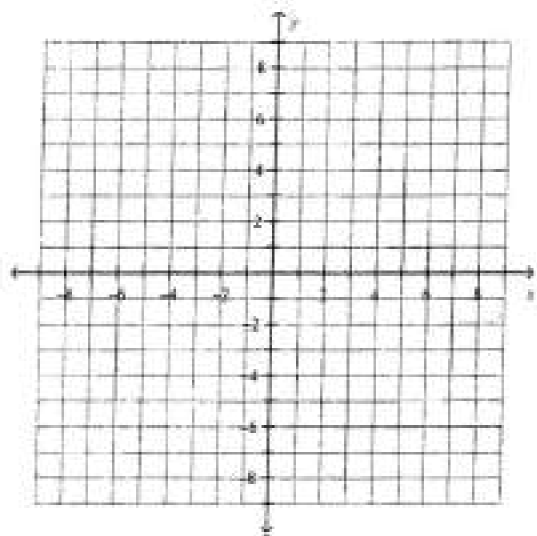
Increasing: ______________

Decreasing: ______________

Positive: ______________

Negative: ______________

$f(-3) =$ ______________



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4. Graph the following function, and answer the corresponding questions.

A. $f(x) = \sqrt{x+3} + 2$

Domain: ______________________

Range: ______________________

1-1?: ______________________

Odd, Even, or neither? ________

x-intercept(s) ______________________

y-intercept(s) ______________________

Intervals for x where the graph is

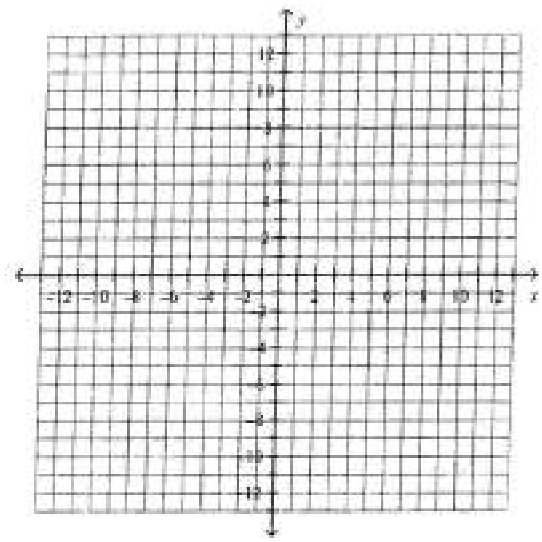
Increasing: ______________________

Decreasing: ______________________

Positive: ______________________

Negative: ______________________

$f(1) =$ ______________________



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Name: ____________________

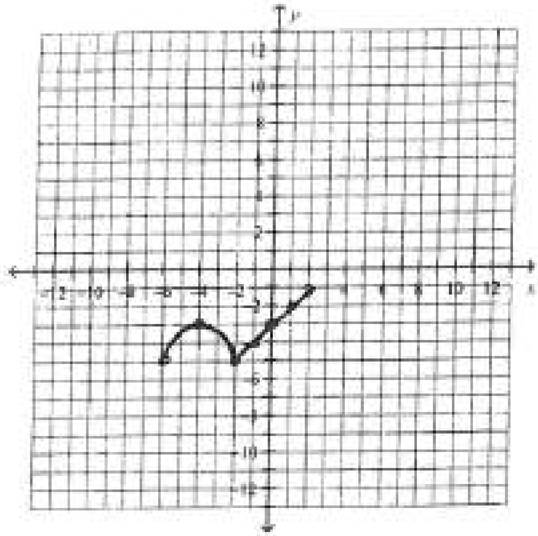
5. Given the graph of $f(x)$ in each case, graph each of the following. Use different color pens or pencils.

A. $f(-x)$ COLOR:_____

$-f(x)$ COLOR:_____

$-f(-x)$ COLOR:_____

$f(x-5)+4$ COLOR:_____

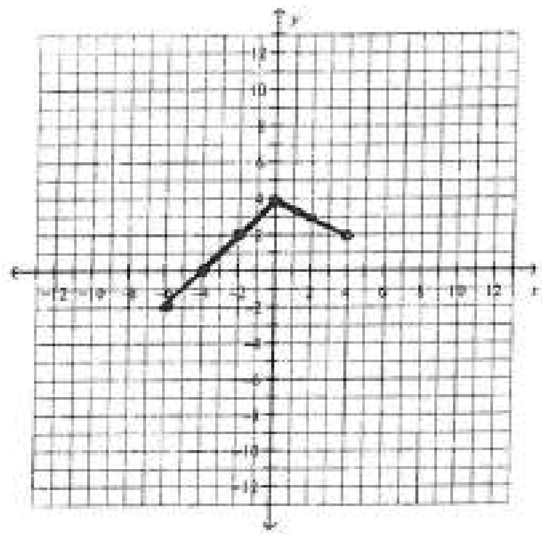


B. $f(2x)$ COLOR:_____

$2f(x)$ COLOR:_____

$f(\frac{1}{2}x)$ COLOR:_____

$\frac{1}{2}f(x)$ COLOR:_____

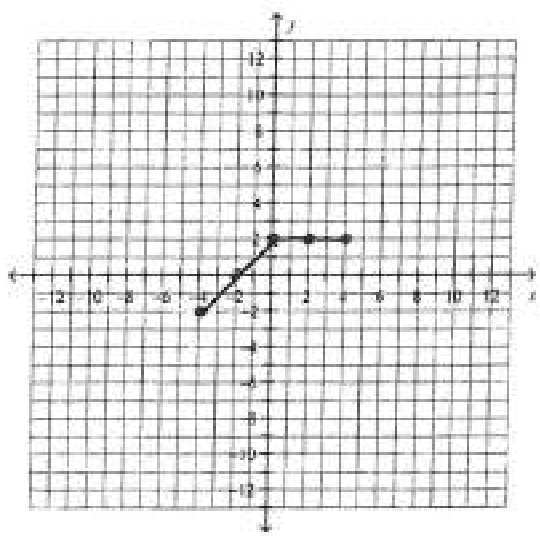


C. $-f(x-3)+4$ COLOR:_____

$f(2x)-5$ COLOR:_____

$2f(x-3)$ COLOR:_____

$f(-x)-3$ COLOR:_____



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Composition of Functions:

A **composite function** is a when a function replaces a variable in another function.

Definition: Let f and g be functions. The function given by $f\left(g(x)\right) = \left(f \circ g\right)(x)$ is called the **composite** of f and g. The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

Try the following examples.

6. Given $f(x) = 3x + 4$, $g(x) = x^2 - 5$, find and simplify completely:

   A. $f\left(g(x)\right)$ B. $f\left(g(2)\right)$

   C. $g\left(f(x)\right)$ D. $g\left(f(2)\right)$

7. Given $f(x) = \sqrt{x-4}$, $g(x) = x^2 + 2x + 5$, find and simplify completely:

   A. $\left(f \circ g\right)(x)$ B. $\left(f \circ g\right)(2)$

   C. $\left(g \circ f\right)(x)$ D. $\left(g \circ f\right)(2)$

8. Given $f(x) = \frac{1}{5}x + 4$, $g(x) = 5x - 20$, find and simplify completely:

   HINT: These are inverse functions.

   A. $f\left(g(x)\right)$ B. $f\left(g(2)\right)$

   C. $g\left(f(x)\right)$ D. $g\left(f(2)\right)$

Models:

Given the following set of data, answer the questions below.

Year(1980=0)	6	7	8	9	10	11	12	13	14
Millions of people in HMOS	25.7	29.3	32.7	34.7	36.5	38.6	41.4	45.2	51.1

9. ______________________ Find a linear model using the linear regression on your calculator. Round to three decimal places.

10. ______________________ Find a quadratic model using the quadratic regression on your calculator. Round to three decimal places.

11. ______________________ Find an exponential model using the exponential regression on your calculator. Round to three decimal places.

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AP CALCULUS AB: SUMMER PACKET TOPIC: EXPONENTIALS AND LOGARITHMS

logarithms: If $b^x = y$ then $\log_b y = x$. $y > 0, b > 0, b \neq 1$

If $e^x = y$ then $\ln y = x$.

$\log_b b = 1$ $\log_b 1 = 0$ $\log_b b^x = x$ $b^{\log_b x} = x$

$\ln e = 1$ $\ln 1 = 0$ $\ln e^x = x$ $e^{\ln x} = x$

Change of Base:

$$\log_b x = \frac{\ln x}{\ln b} = \frac{\log x}{\log b} = \frac{\log_a x}{\log_a b}$$

Expanding(⇒) and Condensing(⇐):

$\log_b (A \cdot B) = \log_b A + \log_b B$ $\ln(A \cdot B) = \ln A + \ln B$

$\log_b \left(\frac{A}{B}\right) = \log_b A - \log_b B$ $\ln\left(\frac{A}{B}\right) = \ln A - \ln B$

$\log_b A^x = x \log_b A$ $\ln A^x = x \ln A$

Write each exponential equation as a logarithmic equation.

1. $2^x = 8$ 2. $(2x)^5 = 8y$ 3. $e^{-3} = 5y$ 4. $e^4 = x$

Write each logarithm equation as an exponential equation.

5. $\log_2 x = y$ 6. $\log(3x) = 8$ 7. $\ln x = 7$ 8. $\ln 5x = y$

Evaluate (round to 3 decimal places on #18-20):

9. $\log_2 16 =$ 10. $\log_3 \sqrt{3} =$ 11. $\log_{(1/5)} 125 =$ 12. $\log_7 \frac{1}{49} =$

13. $\ln e^6 =$ 14. $\ln\left(\frac{1}{e^{44}}\right) =$ 15. $\ln \sqrt[3]{e} =$ 16. $e^{\ln 8} =$

17. $5^{\log_5 8} =$ 18. $\log_2 91 =$ 19. $\log_\pi 27 =$ 20. $\log 200 =$

21. $\log 10000 =$ 22. $\log 0.0001 =$ 23. $\ln\left(\log 10^e\right) =$ 24. $\log\left(\ln e^{1000}\right) =$

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Expand into simpler logarithms:

25. $\log_3\left(9x^4\right)$

26. $\log_2\left(\dfrac{x^4}{y^3}\right)$

27. $\ln\left(\dfrac{e^4x^2}{y^7\sqrt{z}}\right)$

28. $\ln\left(\dfrac{x^5\left(x^2-1\right)^3}{\sqrt[3]{x+5}}\right)$

Condense into one logarithm, and simplify if needed.

29. $3\ln x-4\ln y+5\ln z$

30. $\log\left(x^2-1\right)-\log(x+1)$

31. $4\log_5 x-\frac{1}{2}\left(\log_5(x-7)+\log_5 x\right)$

32. $5+3\ln\left(x^2\right)-4\ln\left(x^3\right)$

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Graphs of exponentials:

$y = b^x$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Asymptote: $y = 0$

y-intercept: $(0, 1)$

If $b > 0$, the function is increasing.
If $0 < b < 1$, the function is decreasing.

Graphs of Logarithms:

$y = \log_b x$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = 0$

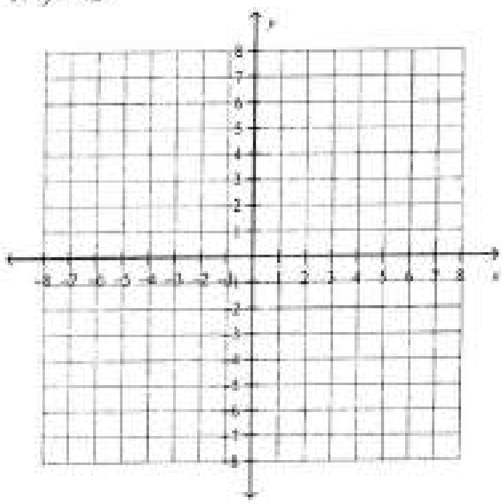
x-intercept: $(1, 0)$

If $b > 0$, the function is increasing.
If $0 < b < 1$, the function is decreasing.

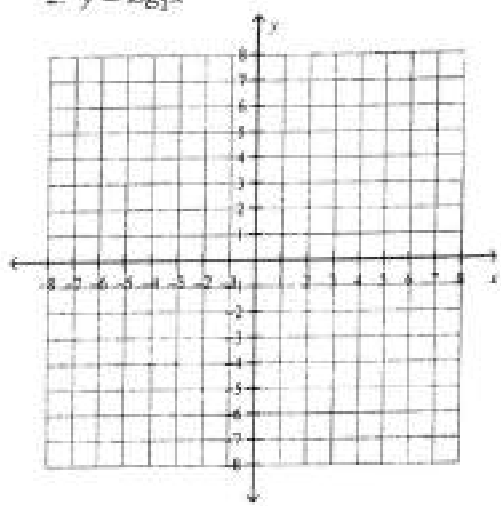
All of these could change based on transformations performed on these parent functions. Exponentials and Logarithms, with the same base, are inverse functions.

Graph:

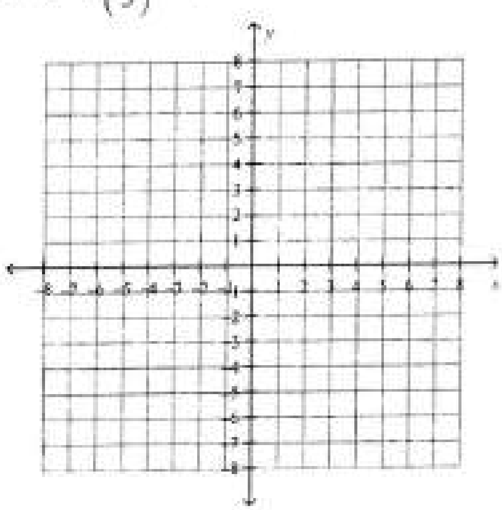
1. $y = 2^x$



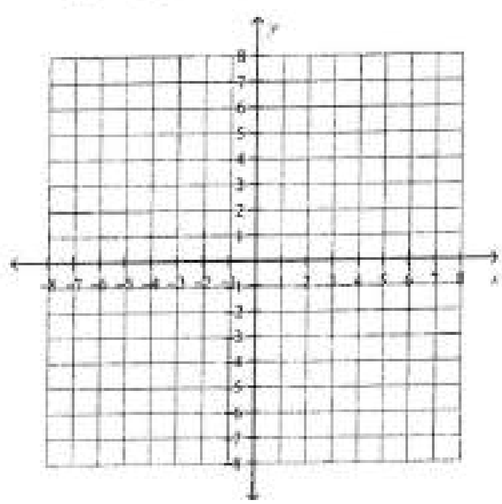
2. $y = \log_3 x$



3. $y = 2\left(\frac{1}{3}\right)^x - 4$



4. $y = \log_3 (x - 4)$



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Name: ____________________

Solving exponential equations. Find the exact answer. Simplify whenever possible.

1. $2^x = 5$

2. $4e^{x-2} = 8$

3. $3^{5x+8} = 9^{2x-4}$

4. $3^{5x+8} = 7^{2x-4}$

5. $2000e^{-0.04t} = 200$

6. $e^{2x} - 5e^x + 6 = 0$

7. $25^x - 2 \cdot 5^x - 8 = 0$

Solving logarithmic equations. Find the exact answer. Simplify whenever possible.

1. $\log_4 x = 3$

2. $\ln(x-2) = 5$

3. $\log(3x-8) = \log(x+6)$

4. $\log_3(x+3) + \log_3(x-5) = 2$

5. $\ln(x) + \ln(x+10) = \ln(4) + \ln(x+4)$

6. $4\ln(4-3x) = 12$

7. $\ln(x+1) - \ln(x-2) = \ln(x+6) + \ln(x-5)$

Name: ____________________

AP CALCULUS AB: SUMMER PACKET TOPIC: TRIGONOMETRY

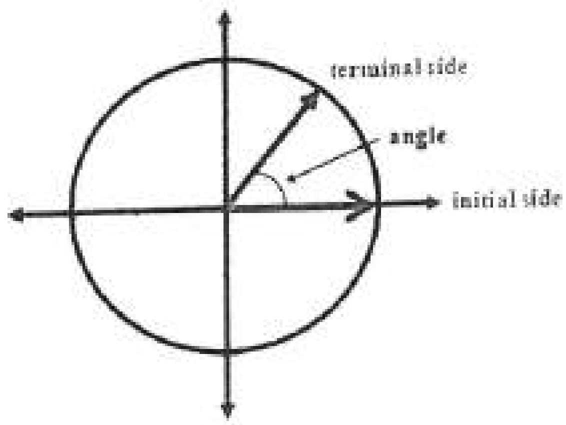
Angles of Rotation:

An **angle** has three parts, an **initial ray**, a **terminal ray,** and a **vertex**. Angles can be measure in **degrees** $\left(\frac{1}{360}\ \textit{of a circle}\right)$ or radians $\left(\frac{1}{2\pi}\ \textit{of a circle}\right)$. To convert from degrees to radians, multiply by $\frac{\pi}{180}$. To convert from radians to degrees, multiply by $\frac{180}{\pi}$.

EX: $60^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{3}$ $45^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{4}$ $-30^\circ \times \frac{\pi}{180^\circ} = \frac{-\pi}{6}$ $360^\circ \times \frac{\pi}{180^\circ} = 2\pi$

EX: $\frac{3\pi}{2} \times \frac{180}{\pi} = 270^\circ$ $\frac{-5\pi}{6} \times \frac{180}{\pi} = -150^\circ$ $\frac{-11\pi}{3} \times \frac{180}{\pi} = -660^\circ$ $\pi \times \frac{180^\circ}{\pi} = 180^\circ$

An angle is in **standard position** if its initial ray lies on the positive x-axis and the vertex is the **origin**. A **counterclockwise** rotation is a **positive angle.** A **clockwise** rotation is a **negative angle.**



Angles are **coterminal** if they share a terminal rays, such as 60° and -300°. The general formula is $\theta^\circ + 360^\circ k,\ k \in Z$ for degrees or $\theta + 2\pi k,\ k \in Z$ for radians.

Try the following examples. If the angle is given in degrees, convert it to radians, and it the angle is given in radians, convert it to degrees. Draw the angle of rotation. Find two coterminal angles to the given angle(same units), one positive, and one negative.

1. A. 120°
   radians:____
   Coterminal
   Pos:______
   Neg:______

   B. -135°
   radians:____
   Coterminal
   Pos:______
   Neg:______

   C. 330°
   radians:____
   Coterminal
   Pos:______
   Neg:______

2. A. $\frac{-7\pi}{6}$
   degrees:____
   Coterminal
   Pos:______
   Neg:______

   B. $\frac{5\pi}{4}$
   degrees:____
   Coterminal
   Pos:______
   Neg:______

   C. $\frac{5\pi}{3}$
   degrees:____
   Coterminal
   Pos:______
   Neg:______

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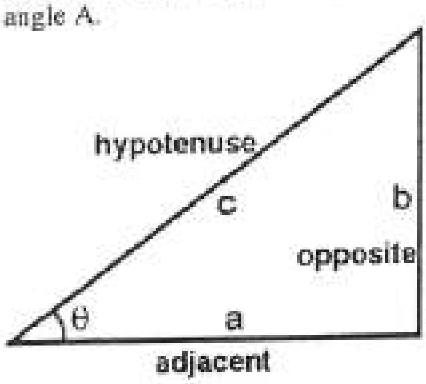
Trigonometric Functions:

In a right triangle ABC, let θ be the acute angle A in the triangle. The side across from angle A, or side BC, is called the **opposite** of angle A. The side across from the right angle C, or side AB, is called the **hypotenuse** and is the longest side in the right triangle. The only side remaining, the side next to angle A, or side AC, is called the **adjacent to** angle A.



Let angle θ be an angle in standard position with the point (x,y) lying on the terminal side of the angle. Draw a circle through the point (x,y). The radius of this circle is $r=\sqrt{x^2+y^2}$, as shown in the following diagram.

Definition of The Six Trigonometric Functions:

Right triangle definition, where $0<\theta<\frac{\pi}{2}$

$$\sin\theta=\frac{opposite}{hypotenuse}\quad \cos\theta=\frac{adjacent}{hypotenuse}\quad \tan\theta=\frac{opposite}{adjacent}$$

$$\csc\theta=\frac{hypotenuse}{opposite}\quad \sec\theta=\frac{hypotenuse}{adjacent}\quad \cot\theta=\frac{adjacent}{opposite}$$

Circular function definitions, where θ is any angle

$$\sin\theta=\frac{y}{r}\quad \cos\theta=\frac{x}{r}\quad \tan\theta=\frac{y}{x}$$

$$\csc\theta=\frac{r}{y}\quad \sec\theta=\frac{r}{x}\quad \cot\theta=\frac{x}{y}$$

Trigonometric Identities:

Pythagorean Identities

$\sin^2\theta+\cos^2\theta=1$

$\tan^2\theta+1=\sec^2\theta$

$1+\cot^2\theta=\csc^2\theta$

Odd-Even Identites

$\sin(-\theta)=-\sin\theta$	$\cos(-\theta)=\cos\theta$	$\tan(-\theta)=-\tan\theta$
$\csc(-\theta)=-\csc\theta$	$\sec(-\theta)=\sec\theta$	$\cot(-\theta)=-\cot\theta$
Odd functions	*Even functions*	*Odd functions*

Supplement Identities

$\sin(\pi-\theta)=\sin\theta$ $\quad\csc(\pi-\theta)=\csc\theta$

$\cos(\pi-\theta)=-\cos\theta$ $\quad\sec(\pi-\theta)=-\sec\theta$

$\tan(\pi-\theta)=-\tan\theta$ $\quad\cot(\pi-\theta)=-\cot\theta$

Sum or Difference Identities

$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\sin\beta\cos\alpha$

$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$

$\tan(\alpha\pm\beta)=\dfrac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$

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Half-Angle Identities

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

Double-Angle Identities

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

Ratio Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Law of Sines

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

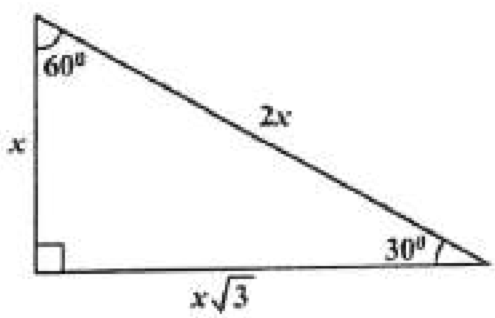
$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Special Right Triangles:

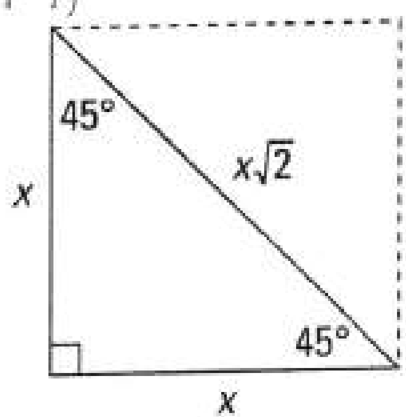
$30° - 60° - 90°$

$\left(\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}\right)$



$45° - 45° - 90°$

$\left(\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}\right)$



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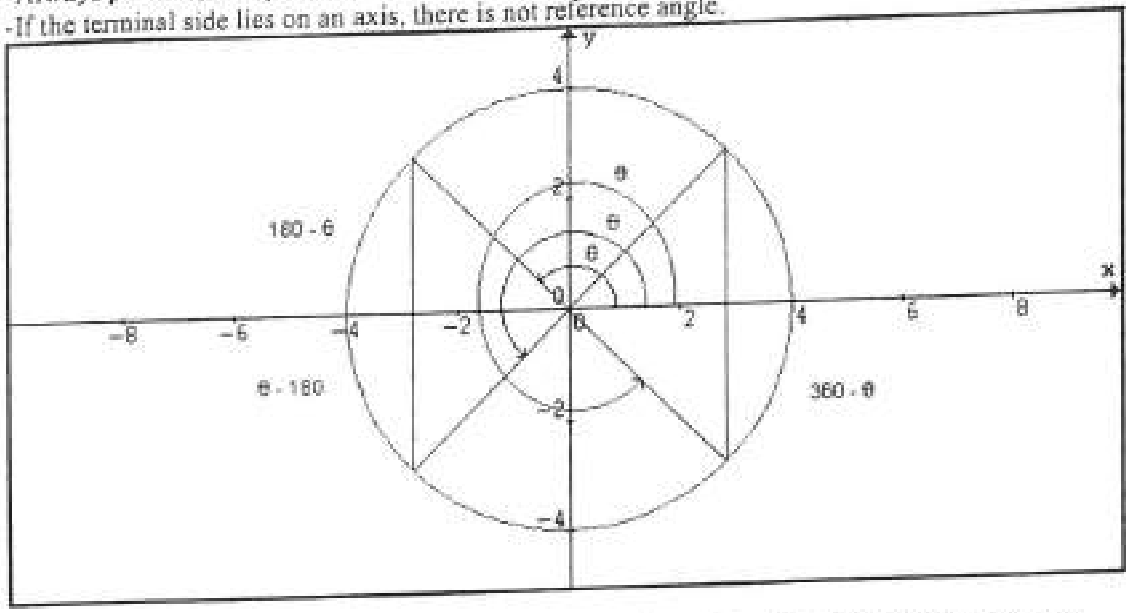
Reference Angles:

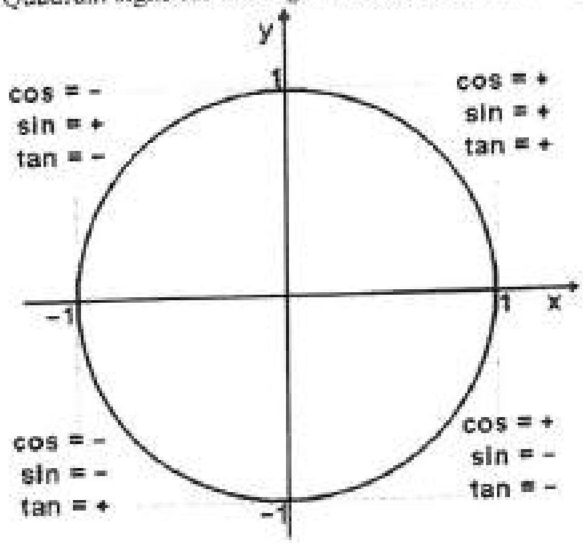
-Always the measure of the angle between the x-axis and the terminal side of θ.

-Always positive, always acute.

-If the terminal side lies on an axis, there is not reference angle.



Quadrant signs for the trigonometric functions. The reciprocal functions follow the same rules.



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Complete the following examples:

3. Find the reference angle for each standard position angle θ.
   A. $35°$ B. $130°$ C. $260°$ D. $-45°$ E. $\frac{7\pi}{6}$ F. $\frac{\pi}{2}$

4. Find the exact value of the following trigonometric functions. No decimals. Simplify completely.

A. $\sin(120°)$

$\cos(30°)$

$\tan(-150°)$

$\cot(300°)$

$\sec(225°)$

$\csc(45°)$

B. $\sin\left(\frac{11\pi}{4}\right)$

$\cos\left(\frac{\pi}{3}\right)$

$\tan\left(\frac{\pi}{6}\right)$

$\cot\left(\frac{-\pi}{6}\right)$

$\sec\left(\frac{-3\pi}{4}\right)$

$\csc\left(\frac{\pi}{3}\right)$

C. $\sin(\pi)$

$\cos(90°)$

$\tan\left(\frac{3\pi}{2}\right)$

$\cot(-90°)$

$\sec(0°)$

$\csc(2\pi)$

5. The coordinate given in each case lies on the terminal side of θ. Find all six trigonometric functions. No decimals. You must simplify your answers.

A. $(3,4)$

$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

$\cot\theta =$

$\sec\theta =$

$\csc\theta =$

B. $(-2,3)$

$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

$\cot\theta =$

$\sec\theta =$

$\csc\theta =$

C. $(0,6)$

$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

$\cot\theta =$

$\sec\theta =$

$\csc\theta =$

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6. Given that $\sin\theta =$ [illegible], find the following:

A. $\sin(-\theta)$ B. $\sin(\pi - \theta)$ C. $\csc\theta$ D. $\sin(2\theta)$

7. Find the exact values of the following.

A. $\cos(75°)$

B. $\sin(15°)$

C. $\tan(75°)$

8. Solve the equation for θ(in radians): $\sin\theta = \frac{-\sqrt{3}}{2}$

9. Solve the equation for θ(in degrees): $2\cos^2\theta = 1$

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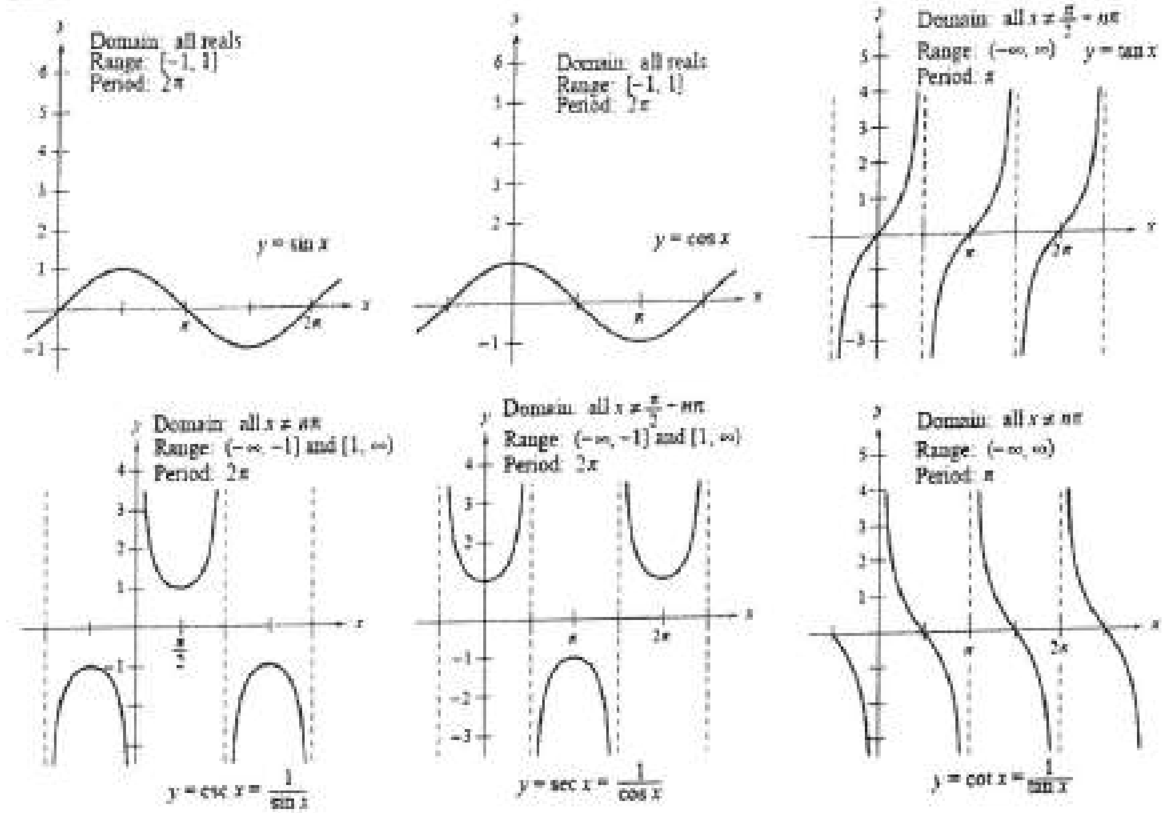
Graphs of Trigonometric Functions:

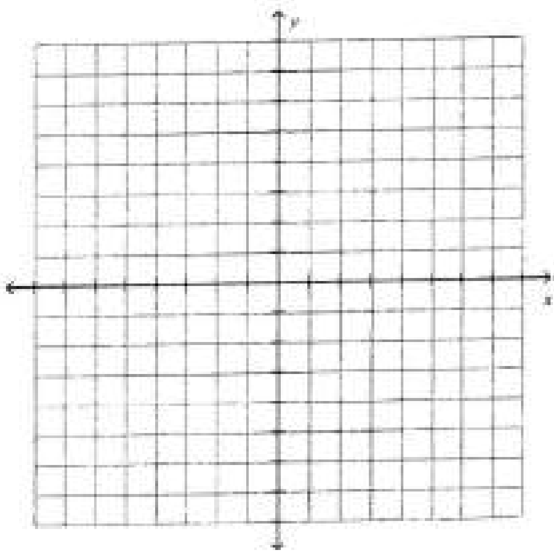


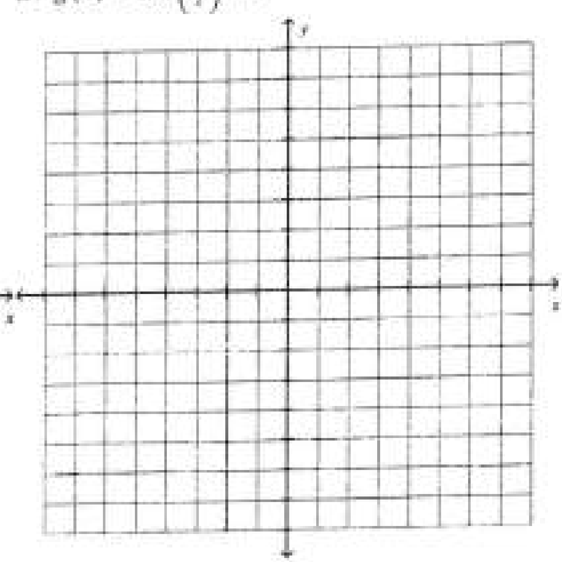
The graphs of the six trigonometric functions

10. Each graph should show at least two cycles of the function.

A. Graph: $f(x) = 3\sin(2x) + 4$

B. $g(x) = \sec\left(\frac{x}{2}\right) + 3$





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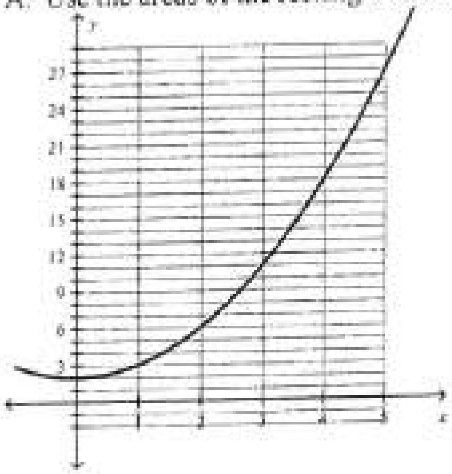
AP CALCULUS AB: SUMMER PACKET   TOPIC: AREAS AND TANGENTS

Numeric Response

1. Below is the graph of $y = x^2 + 2$.

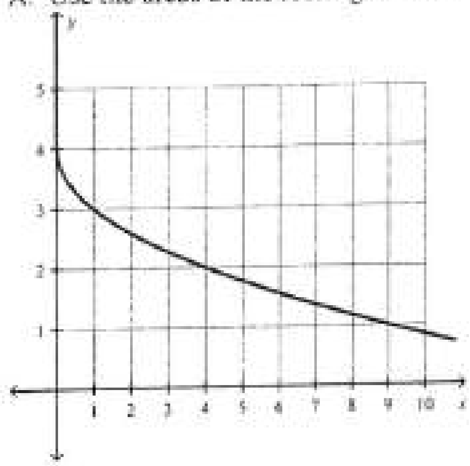
   A. Use the areas of the rectangles drawn to estimate the area under the curve from 0 to 5.



   B. Is the estimation greater than or less than the actual area under the curve?

2. Below is the graph of $y = 4 - \sqrt{x}$.

   A. Use the areas of the rectangles drawn to estimate the area under the curve from 0 to 9.



   B. Is the estimation greater than or less than the actual area?

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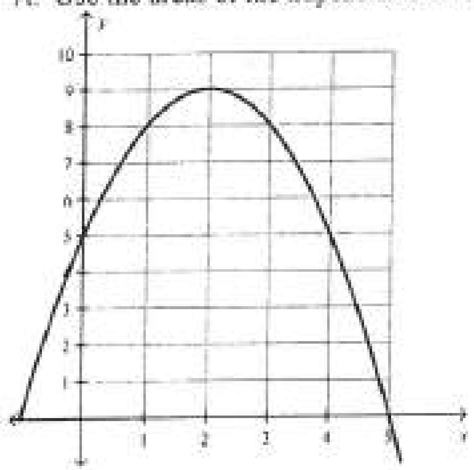
3. Below is the graph of $y = -x^2 + 4x + 5$.

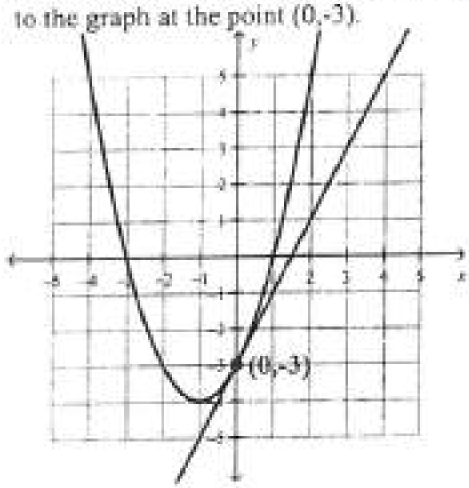
A. Use the areas of the trapezoids drawn to estimate the area under the curve from 0 to 5.



B. Is the estimation greater than or less than the actual area?

4. Below is the graph of $y = x^2 + 2x - 3$.

A. Find the equation of the line tangent to the graph at the point (0,-3).



B. Find the equation of the line tangent to the graph at the point (-1,-4).